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Public R&D investment and the Labour Share*

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Abstract

This paper studies the effects of public investment in R&D on the labour income share in the US and in a pool of selected advanced economies. Empirical results show that an increase in public investment in R&D produces a substantial decline in the labour income share in the short run together with the standard long run positive effects in terms of output, investment and consumption. This means that public investment in R&D can generate non-negligible (re)distribution effects in the short run that can potentially harm agents that are heavily dependant on labour income. In the second part of the paper I try to rationalise this result through the lens of a DSGE model with endogenous technology adoption. In the model a government subsidy towards adoption of new technologies induces firms to substitute resources away from capital accumulation and towards the R&D process. This initial decline in capital accumulation generates a decline in hours worked consistent with complementarity in production between capital and labour. This in turn drives down labour income that, together with the increase in output, induces the fall in the labour share.

JEL classification: E32, E62, C52.
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1 Introduction

Recent empirical evidence shows that factor income shares[1] are significantly less stable than historically considered since the work of Kaldor (1957). This instability in the distribution of national income between workers and owners of capital is true both in the long run and in the short run and has attracted renewed attention from economists (Karabarbounis & Neiman (2014), Piketty (2014), Ríos-Rull & Santaeulalia-Llopis (2010), Cantore, León-Ledesma & Ferroni (2018), Cantore, Freund & Melina (2018)). At the same time the increase in wealth and income inequality across the world has generated renewed attention on the redistributive effects of policy. However so far there is little empirical evidence on the effects of fiscal policy on the labour income share and, to the best of my knowledge, no evidence on the effects of the different components of government expenditure on the redistribution of income.

The aim of this project therefore, is to analyse the effects of different components of public spending on the distribution of income across the business cycle with special focus on public spending in research and development (R&D). First I present empirical evidence using a conventional SVAR model to identify how different components of fiscal spending have affected the distribution of income in the US in the last decades. The main focus on the US economy is justified by the richness of data on the components of government spending at quarterly frequencies, where is possible to distinguish between: government expenditure in consumption of goods and services and government investment. Most importantly, given the focus of this study, I will look at one specific category of government investment which is the amount of R&D financed by the government.

Results shows that the responses of output, consumption, investment, the labour share and the long run interest rate are almost indistinguishable when looking at an increase in total government expenditure or in one of its two components (government consumption and investment). As usually found in the literature, a fiscal expansion boosts output and consumption, crowds out private investment and increases government revenues via a tax increase. Crucially however, when I identify an increase in government expenditure in R&D I find that the responses of private investment and the labour income share are of opposite sign compared to the effects of other fiscal shocks just described. Private investment increases while the labour income share declines. The boost in private investment produces a strong and long lasting positive effect on output in line with the literature on the effects of R&D on economic growth. But the negative, albeit temporary, effect of an increase

[1] The proportion of gross value added going back to each production inputs.

in R&D spending on the labour share points towards a redistribution effect that favours capital income at the expenses of labour income. The empirical analysis also presents cross country results showing that that these responses seems robust across advanced economy, looking at annual data for Canada, the UK, France and Germany. However, using only SVAR evidence, it is not possible to give a structural interpretation on the transmission mechanism of the identified shock to public R&D spending.

In order to rationalize these results, in the second part of the paper I present a DSGE model that can account for these stylised facts. The model includes exogenous unproductive government spending that usually feature these models but it also allows for an explicit government investment in R&D. This is introduced via a process of invention and adoption of intermediate goods in production by extending the model of Comin (2009) as in Cantore & Swarbrick (2018). Fiscal policy then consists of a combination of two instruments: public investment in the R&D process (productive spending) and government consumption (unproductive spending). The model, calibrated to the US economy, is then used to shed light on the transmission mechanism of a government subsidy to R&D on investment and the labour income share as identified in the first part of the paper.

Simulations results show how a public subsidy to the R&D process made by private firms in the model economy is able to generate simultaneously the long run positive effects on output, investment and consumption together with the short run decline in the labour share. The mechanism that generates the decline in the labour share depends on the substitutability in production between capital and adoption of new technologies together with the complementarity between capital and labour. A government subsidy to R&D induces firms to shifts resources from the accumulation of capital towards the adoption of new technologies. Basically the subsidy generates an increase in the efficiency and profitability of the R&D process. The consequent decline in capital accumulation generates a decline in hours worked given their complementarity with capital in the production process of firms. This in turn generates a decline in labour income that induces the fall in the labour share.

While, to the best of my knowledge, there is not a similar study on the effects of a shock to public investment in R&D at short and medium run frequencies, this paper is related to two main strands of the literature. First of all, to the macroeconomic literature on R&D. Most of this literature has fucussed on the long run effects of R&D on growth and well being (see Romer (1990), Jones (2004) and references therein). Altough quite recently there has been an increasing interest on the cyclicality of R&D (see Barlevy (2007), Artuç & Pourpourides (2014) and Anzoategui et al. (2016)), none of these studies focussed on the cyclicality of public investment in R&D.

On the other hand, this paper is related to the very prolific litarture on the cyclical effects of fiscal policy (see Blanchard & Perotti (2002), Ramey (2011) and Ramey (2016) for a comprehensive survey). Once again I am not aware of any study that focussed on the implication of fiscal policy via public investment in R&D. This paper is however closely related to Cantore, Freund & Melina (2018), which is a comprehensive cross-country study on the effects of fiscal policy on the labour share. While some of their data is also used here there is a clear distinction in the focus and empirical approach between the two. Here the aim is mainly on the relationship between public R&D investment and the labour share while they study the effect of total government spending on the labour income share. This difference in the focus between the two studies is also reflected in the empirical approach. Cantore, Freund & Melina (2018) present, for the US, results using all the state of the art identification techniques in order to deal with the issues of fiscal foresight and non-fundamentalness. Here this is not possible given the lack of either forecasts or instruments data on public spending in R&D.

The rest of the paper is structured as follows. Section 2 presents the data, the econometric framework and the empirical results. Section 3 presents that theoretical model, it's calibration and simulation by focusing on the transmission mechanism of the government subsidy in R&D. Finally section 4 concludes.

2 Empirics

The focus of this section is to establish the causal effect in the data of an unanticipated government spending shock on output, consumption, investment and the labour share. While most of the analysis focuses on the United States, the only country with a good sample of data on government expenditure in R&D at quarterly frequencies, I report some evidence also on Canada, the United Kingdom, France and Germany. The empirical analysis will be based on a SVAR with recursive identification a la Blanchard & Perotti (2002). This type of identification has been criticised in the literature for not being able to distinguish between unanticipated and anticipated shocks. This is due to the fact that changes in fiscal expenditure might be anticipated by economic agents even if they cannot be predicted by the econometrician estimating the SVAR (Ramey 2011). This issue known as *fiscal foresight* is usually dealt in the literature by either exploiting truly exogenous and unanticipated shocks to defence spending as in Ramey (2011) and Ben Zeev & Pappa (2017) or by using data on agents forecasts of public spending to *purify* fiscal shocks from their anticipated (*news*) component (see Forni & Gambetti (2016) for details).

These approaches to deal with *fiscal foresight* however, have been developed to deal with anticipation issues for total government spending.

In the context of public R&D spending it seems unreasonable to use narrative measures of military spending. On the other hand surveys of professional forecasters do not include information specific on government R&D spending. Hence the recursive identification approach of Blanchard & Perotti (2002) remains the only viable solution for a SVAR that wants to analyze shocks to public R&D investment. This approach basically assumes that government investment in R&D is predetermined within the quarter so a shock to public R&D spending can be identified using a standard Cholesky decomposition with public R&D spending ordered first. This means that all other variables included in the SVAR are in principle allowed to have an instantaneous response following a change in public R&D. While public spending in R&D does not respond contemporaneously to all the macroeconomic shocks hitting the other variables.[2]

2.1 Data: construction and sources

Our data set consists of real output, consumption, investment, government receipts, the 10-year real interest rate, the labour share and different components of government spending. It follows very closely the data construction and baseline specification in Cantore, Freund & Melina (2018).[3]

For the USA all the series are taken from the NIPA tables[4] of the Bureau of Economic Analysis with the exception of the 10 year interest rate[5]. For Canada, the UK, France and Germany all series except the 10 years interest rate, are taken from the national accounts of the OECD. For these countries quarterly decomposition of government expenditures with a long time span are not available so I rely on annual data from the OECD MSTI database on the percentage of gross domestic expenditure on R&D financed by the government. Data on the 10 year interest rate for all the countries are retrieved from the FRED database of the Federal Reserve Bank of St. Louis.[6]

[2]This assumption seems more reasonable when applied to R&D as opposed to total government spending given that fact that R&D spending is more persistent and less prone to be affected by political cycles and legislation changes.

[3]So I remind the interested reader to that paper and its appendix.

[4]Quarterly data on Government consumption expenditures and Gross investment which include government investment in R&D are taken from NIPA table 3.9.5.

[5]Cantore, Freund & Melina (2018) show how the inclusion of the long run interest rate in the SVAR information set reduces substantially the forecasteability of government spending shocks.

[6]All national income series are seasonally adjusted by the source and, unless otherwise stated, are

The series for the labour share deserves special attention. Measuring the share of labour in total income is more complicated by problems associated with how to impute certain categories of income to labour and capital owners. The existence of self-employment income, the treatment of the government sector, the role of indirect taxes and subsidies, household income accruing from owner occupied housing, and the treatment of capital depreciation, are common problems highlighted in the literature. These have been discussed at length in Gollin (2002), Gomme & Rupert (2004) and McAdam et al. (2015). Here for the US I use a measure of the labour share constructed using data extracted from the US NIPA tables following a comprehensive approach to dealing with "mixed" income (such as that of the self-employed) proposed by Cooley & Prescott (1995) and further developed by Gomme & Rupert (2004) and Ríos-Rull & Santaeulalia-Llopis (2010). Crucially both Cantore, León-Ledesma & Ferroni (2018) in the contest of a SVAR identified for monetary policy and Cantore, Freund & Melina (2018) on a SVAR for fiscal policy have shown that other measures the labour share constructed with different approaches have very similar properties at business cycle frequencies. For the rest of the countries I follow again Cantore, Freund & Melina (2018) and compute the labour share as compensation of employees over gross value added at factor cost.

2.2 Econometric Methodology

As a baseline specification for the estimation of different types of government spending shocks I consider a 5 variable SVAR estimated for the United States using quarterly data spanning form 1948:Q1 to 2007:Q4. The starting date is dictated by data availability while the end date is chosen to be before the Great Recession to avoid potential structural breaks.[7] The variables in the information set are: (i) the log of a real component of government expenditure; (ii) the log of real net taxes; (iii) the log of real GDP; (iv) the log of the labour share; and (v) the 10-year real interest rate. Regarding (i) I will consider in turn: (a) total government spending (government consumption + investment); (b) government consumption; (c) government investment; and (d) government investment only in R&D.

I assume that the joint co-movements of these macroeconomic variables can be

deflated using the GDP deflator. For all series except for the interest rate, I take the natural logarithm and multiply the resulting series by 100, yielding the series used in the estimation. Where necessary I take the arithmetic average of monthly figures to obtain quarterly series and the arithmetic average of quarterly figures to obtain annual series.

[7] Results using the full sample however show similar results as shown in figure 3.

described by a VAR of order p which takes the following form:

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + ...\Phi_p y_{t-p} + e_t \quad e_t \sim N(0, \Sigma),$$

where y_t is a vector that contains the observable variables and ϵ_t is a vector of normal zero mean i.i.d. shocks with $\Sigma = E(\epsilon_t \epsilon_t')$. $\Phi_0, \Phi_1, ..., \Phi_p$ are matrices of appropriate dimensions describing the dynamics of the system. The reduced form VAR is compatible with several structural representations where reduced form residuals can be expressed as linear combination of structural uncorrelated innovations, i.e.

$$e_t = \Omega \nu_t,$$

where $\Omega\Omega' = \Sigma$ and $E(\nu_t \nu_t') = I_n$. I follow Blanchard & Perotti (2002) to retrieve the fiscal shocks from the rotation matrix, Ω.

I identify a fiscal shock using a Cholesky recursive ordering.[8] The Cholesky ordering follows the identification assumption that a shock to government spending can have an instantaneous effect on all other variables. This implies also that government spending does not respond contemporaneously to all the macroeconomic shocks hitting the other variables.

In line with standard Bayesian practice, the SVAR is estimated using Markov Chain Monte Carlo Methods employing a normal-diffuse ("Jeffrey's") prior for the coefficient matrix and the covariance matrix of the reduced-form innovations, respectively. Impulse responses and posterior credible sets are generated based on 10,000 draws. The lag length is chosen based on information criteria, which suggest the use of two lags. The equations are estimated in levels to preserve potential cointegrating relationships among the variables. I also include a quadratic time trend as in Ramey (2016) to capture features such as the productivity slowdown or the effect of the baby boom.

2.3 Results

Figure 1 depicts the impulse responses to a surprise government spending shock, for each of the 4 different measures of government spending. From top to bottom I present a shock to: total government spending, government consumption, investment and R&D. In all figures reporting empirical impulse responses, solid lines represent the median response while dotted lines represent the 16th and the 84th percentile, unless otherwise indicated.

This picture shows how the response in the first three rows are almost indistinguishable. A positive shock to total spending, government consumption or investment had a significantly positive and persistent effect on output and taxes and a

[8] The order follows exactly the same order as in the information set above.

positive and short lived effect on the long run interest rate. What is changing is the response of the labour share. While a shock to total spending and government consumption has a positive and significant effect for the first few quarters a shock to government investment has a negative and significant effect on the labour share, albeith only on impact.[9] This is a first sign of how government spending components might have a very different impact on the redistribution of income.

The final row looks at a specific component of government investment, public investment in R&D,[10] and produces results quite different from the previous ones. First of all the identified shock looks much more persistent. While for the previous three after 25 quarters the shock has almost fade out, for public R&D spending the median response is still above the effect on impact. This is also reflected in the very persistent response of taxes and GDP and it is in line with the expectation that R&D spending is more persistent than other government purchases and investment and has longer cycles. The real interest rate jumps straight away as opposed to the hump shaped response produced before. Finally, and most importantly for the scope of this work, the labour share declines on impact and remains significantly negative for at least 8 quarters.

Moreover for all the shocks the percentage deviation of the labour share at peak is almost the same as for GDP showing that these effects are economically significant as well.

After establishing this different behaviour of the labour share in response to shocks to different components of government spending, the next step is to check if these are robust to the inclusion of other variables in the information set. Hence I extend the SVAR to a 7 variables one adding private consumption and investment to the picture. Figure 2 presents the results of this exercise. The inclusion of these two variables is also motivated by the fact that the behaviour of consumption and especially investment will be crucial in understanding the transmission mechanism of these shocks to the labour share.

The shocks to total spending and public consumption show similar responses as before as well as showing a positive and hump shaped effect on private consumption and a crowding-out of investment. These results are in line with what is usually found in the literature. The labour share response to the total spending shock is now positive and extremely persistent. Turning to a shock to government investment I can see how it seems that this is the driver to such behaviour of the labour share. Hence adding these two extra variables changes the sign, significance and persistence

[9] Cantore, Freund & Melina (2018) find a similar result.

[10] Government investment in the NIPA table 3.9.5 can be decomposed in: structures, equipment and intellectual property products (IPP). In turn, IPP can be further decomposed in software and R&D.

of the labour share to a government investment shock. Looking instead at the last row of this figure one can see however that the negative and persistent response of the labour share to a public R&D spending shock is robust to the inclusion of private consumption and investment to the SVAR info set. So the take over from this analysis of quarterly US data is that the impact of a shock to public spending in R&D on income (re)distribution looks very different to a shock to any other component of public spending. As a robustness exercise Figure 3 shows as this result is robust to the inclusions of the Great Recession in the SVAR sample.

2.4 Cross country evidence

In this section I present further evidence regarding this *redistributive* effects of a shock to public spending in R&D in a pool of developed economies. Using annual data from the MSTI OECD dataset I estimate a similar SVAR as before for Canada, the United Kingdom, France and Germany. The choice of the countries is due to the availability of at least 30 years of data on the labour share and the percentage of gross domestic expenditure on R&D financed by the government. The sample period is 1981 to 2012 for all the countries except for Canada where it goes from 1983 to 2015. The SVAR is estimated as before including a quadratic trend and 2 lags.

Figure 4 shows that the estimated shock to public R&D is still very persistent in all the countries (a period in the figure corresponds to a year now). It is possible to observe as well a quite heterogeneous response of taxes and GDP across countries where in Canada I observe a positive and persistent effect on GDP like in the US but a quite substantial decline in government receipts as well meaning that public spending on R&D seems to be primarily financed with deficit in this country. In Germany and the UK I observe a positive but less persistent effect on output with a substantial increase in taxes. In France public spending in R&D does not seem to have a huge impact on GDP and government receipts in the short run.

Looking at the labour share one can see a much more homogeneous picture for all the countries expect perhaps the UK. The share of output going to labour declines and stays below trend for quite some time after an increase in government spending in R&D. In Canada for example, the labour share is still below trend after 7 years from the shock. In France, after an initial increase on impact, it declines and stays below trend up to 6 years after the shock. In Germany the effect is observed only in the first 3 years from the shock while the UK is the only country in the sample for which I observe a positive impact from years 6 to 9. Of course the differences in responses between this countries could be due to the substantial heterogeneity between these economies which cannot be captured very well at annual frequencies. However, the substantial homogeneity in the response of the labour share is striking.

3 The Model

In this section I present a Real Business Cycle (RBC) model with endogenous growth that will be used to rationalize the stylized facts presented in previous section. I do so by adding the simple model of Comin (2009)[11] to an otherwise standard RBC model.

In order to provide a structural explanation to the shock on public spending in R&D identified in section 2, I include two types of government spending in this model. Usually in DSGE literature total government spending is modelled as an exogenous AR(1) process that enters in the resource constraint of the economy. The model presented here features this shock which represents *unproductive* government spending. Moreover I assume that the government can direct some of its resources towards *productive* spending by encouraging R&D activities with a subsidy on technology adoption. Cantore & Swarbrick (2018) show that this shock can generate interesting dynamics in a similar model that studies the evolution of the skill premium over the medium run.

As standard in the RBC literature the economy consists of a representative agent that makes consumption and investment decisions and rents capital and labour to a representative firm who produce the only good in the economy. Government is assumed to raise lump sum taxes in order to finance its two types of spending and balances its budget every period. This model adds an endogenous pace of technology adoption to the expanding variety model of technological change pioneered by Romer (1990). Comin (2009) and Anzoategui et al. (2016) provide empirical motivation for the inclusion of technology adoption in models of endogenous technical change.

The source of non-stationarity in the model will be intermediate goods (*materials*) used in production, which in turn will depend on two endogenous variables: the *stock of invented technologies* via R&D and their *speed of adoption*. In order to stationarize the model I follow Cantore & Swarbrick (2018) and borrow from the sticky wages literature and introduce labour packers. A labour packer purchases hours from households at the competitive wage rate and combines it with materials using a Cobb-Douglas technology. Labour packers then sells *efficiency labour* units at *efficiency wage* rates to firms.[12]

[11]This is a simplified version of Comin & Gertler (2006).

[12]This assumption simplifies the computation of the stationary equilibrium of the model but simulations using materials directly into de production function as in (Anzoategui et al. 2016) show that our

3.1 Household

The representative household maximise the expected utility

$$\max_{\{C_t, H_t, K_t\}} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k U\left(C_{t+k}, H_{t+k}\right) \tag{3.1}$$

where β is the discount factor and C_t is consumption. Maximisation is subject to a budget constraint

$$Y_t = W_t H_t + R_t K_{t-1} + \Pi_t - T_t. \tag{3.2}$$

where Y_t is aggregate production, and *end-of-period* capital (K_t) accumulates following:

$$K_t = I_t + (1-\delta) K_{t-1}.$$

The gross return on capital is $R_t = r_t - 1 + \delta$ where r_t is the rental rate and W_t is the wage paid to labour (H_t) .

Moreover households are assumed to own the representative firm and the technology adopters (section 3.2.1) and receive dividends from both summing up to Π_t. Finally they pay lump sum taxes T_t raised by the government to finance public spending in R&D (details in section 3.2.2) and in *unproductive* government purchases.

This leads to the following first order conditions associated with the households problem:

$$\frac{U'(H_t)}{U'(C_t)} = -W_t, \tag{3.3}$$

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1}\left(r_{t+1} + (1-\delta)\right)\right] \tag{3.4}$$

$$\tag{3.5}$$

where $\Lambda_{t-k,t} \equiv \beta U'(C_t)/U'(C_{t-k})$ is the stochastic discount factor.

3.2 Research and Development

The source of endogenous growth in the model is labour-augmenting production materials, M_t, which are produced competitively by combining intermediate goods m using

$$M_t = \left(\int_0^{A_t} m_{i,t}{}^{1/\theta}\right)^{\theta} \tag{3.6}$$

results are not driven by it.

This leads to a relative demand schedule for each intermediate good given by $m_{i,t} = M_t \left(P_{i,t}/P_t^M\right)^{\theta/(1-\theta)}$. θ will be shown to be the mark-up charged by producers of the intermediate good i. It takes one unit of output to produce a unit of intermediate goods, and there are a total of A_t units available for production at time t. A_t represents adopted technologies and is the source of non-stationarity in the model as A_t is increasing over time. The invention and use of intermediate goods is a two stage process. First, the intermediate goods are invented via research and development programmes, and secondly adopted into production.

3.2.1 Adopters

Letting Z_t be the stock of invented technologies, and A_t is the number of technologies currently adopted. A proportion $1-\phi$ of adopted technologies become obsolete every period, and a fraction p_t of non-adopted technologies $Z_{t-1} - A_{t-1}$ become available at the end of period t. A_t then evolves according to

$$A_t = p_t \phi \left[Z_{t-1} - A_{t-1}\right] + \phi A_{t-1} \tag{3.7}$$

where

$$p_t = p\left(\Gamma_t x_t\right) \tag{3.8}$$

$p' > 0$ and $p'' < 0$, x_t is the resources directed to technology adoption, and Γ_t is exogenous to the adopter and defined to ensure consistency with a balanced growth path.

Before turning to the innovation process, I complete the definition of adoption by specifying the solution to the objective function of the adopters. The adopters can adopt an invention and sell to intermediate good producers under monopolistic competition. First, let v_t denote the value to the adopter successfully bringing an innovation into production. This is the simply the present discounted value of future profits

$$v_t = \pi_t + \phi \mathbb{E}_t \left[\Lambda_{t,t+1} v_{t+1}\right] \tag{3.9}$$

If unsuccessful, the adopter can try again in the following period. Let w_t be the value of acquiring a non-adopted innovation, the adopter purchases the innovation x_t to solve

$$w_t = \max_{x_t} \left\{-x_t + \phi \mathbb{E}_t \left[\Lambda_{t,t+1} \left(p_t\left(\Gamma_t x_t\right) v_{t+1} + \left[1 - p_t\left(\Gamma_t x_t\right)\right] w_{t+1}\right)\right]\right\} \tag{3.10}$$

Note that v_t is the present value an adopted technology, but independent of the resources used to get the technology adopted, x_t. Therefore, the resulting first order condition is simply

$$1 = \phi \mathbb{E}_t \left[\Lambda_{t,t+1} p_t' \left(\Gamma_t x_t \right) \left(v_{t+1} - w_{t+1} \right) \Gamma_t \right] \tag{3.11}$$

In setting the price for the intermediate good, adopters maximise revenue $P_{i,t} m_{i,t} - m_{i,t}$ subject to demand $m_{i,t} = M_t \left(P_{i,t} / P_t^M \right)^{\theta/(1-\theta)}$, leading to $P_{i,t} = \theta$. By symmetry, $M_t = A_t m_{i,t}$, and so it follows

$$P_t^M = A_t^{\frac{1-\theta}{\theta}} \theta. \tag{3.12}$$

3.2.2 Government investment in R&D

Here I follow Cantore & Swarbrick (2018) and introduce a source of exogenous variation in Comin (2009)'s model by assuming that the policymaker encourages R&D activities by subsidizing adoption. Specifically, the adopted technology sells for price $P_{i,t}$ but the adopter receives $(1 + \tau_t) P_{i,t}$. The solution to the adopter problem then is modified accordingly:

$$P_t^M = A_t^{\frac{1-\theta}{\theta}} \frac{\theta}{1 + \tau_t}. \tag{3.13}$$

τ_t will be modelled as an exogenous AR(1) process and will be interpreted as a government policy subsidizing R&D in the same vein in which exogenous government spending is usually modelled. The crucial difference here is that this does not represent *wasteful* government spending, which I'll introduce below, but a subsidy to innovation.[13]

3.2.3 Innovators

The stock of intermediate goods invented by innovator p, that are non-obsolete, follows the law of motion

$$Z_t(p) = \varphi_t S_t(p) + \phi Z_{t-1}(p) \tag{3.14}$$

where S_t are resources innovator p dedicates to the process, and φ_t the productivity of innovation determined by the aggregate state of the economy:

$$\varphi_t = \chi Z_{t-1} \left(\frac{S_t}{K_t} \right)^{\rho - 1} (K_t)^{-1} \tag{3.15}$$

The solution to the innovator problem leads to a zero profit condition

$$1 = \varphi_t \mathbb{E}_t \left[\Lambda_{t,t+1} w_{t+1} \right] \tag{3.16}$$

[13]Technically this shock can also be interpreted as a shock to the mark-up in the market for materials.

where w_t is the value of an innovation to an adopter, that is, the price at which the technology can be sold. Substituting (3.14) into (3.16) yields

$$S_t = \mathbb{E}_t \left[\Lambda_{t,t+1} w_{t+1} \left(Z_t - \phi Z_{t-1} \right) \right] \tag{3.17}$$

where the innovator index is dropped due to symmetry. In aggregate, the stock of available technologies follows the law of motion

$$Z_t = Z_{t-1} \left[\chi \left(\frac{S_t}{K_t} \right)^{\rho} + \phi \right] \tag{3.18}$$

3.3 Labour Packers

Competitive labour packers purchase labour from the households, combine with labour-augmenting technology M_t, bought at price P_t^M at the end of the previous period, and sell to firms as labour services. The following technology is used to produce efficiency labour units

$$L_t = M_{t-1}^{\eta} \left(H_t \right)^{1-\eta} \tag{3.19}$$

Profit maximisation implies that the wage rate, W_t, is a linear combination of technology and the efficiency wage rate:

$$W_t = \hat{W}_t \left(1 - \eta \right) \left(\frac{M_{t-1}}{H_t} \right)^{\eta} \tag{3.20}$$

Demand for technology (materials) must then satisfy:

$$\frac{\eta}{1-\eta} \mathbb{E}_t \left[\Lambda_{t,t+1} W_{t+1} H_{t+1} \right] = \mathbb{E}_t \left[\Lambda_{t,t+1} P_t^M M_t \right]. \tag{3.21}$$

Hence M_t can be interpreted as labour-augmenting technology that improves productivity of labour and $\eta < 1$.

3.4 Firms

Firms combine labour with capital using the following technology

$$Y_t = f \left(Z_t^k, K_{t-1}, Z_t^l, L_t; \theta^y \right) \tag{3.22}$$

where f is a CES aggregator defined below and Z_t^k and Z_k^l are technology shocks augmenting capital and labour respectively. θ^y is a vectors of parameters. Hence the problem of the firm is to minimize the costs of inputs K_t and L_t subject to (3.22).

With perfect capital and labour markets, demand for capital and labour must satisfy

$$\hat{W}_t = f' \left(L_t \right) \tag{3.23}$$

$$r_t = f' \left(K_{t-1} \right). \tag{3.24}$$

3.5 Aggregation, preferences and technology

The model is closed with the aggregate resource constraint

$$Y_t = C_t + I_t + S_t + x_t\left(Z_{t-1} - A_{t-1}\right) + T_t \tag{3.25}$$

where T_t represents government receipts from lump sum taxes levied on households. I assume that the government balances his budget and uses tax receipts to finance exogenous *unproductive* government spending G_t and the subsidy to R&D adoption every period:

$$T_t = \tau_t P_t^M M_t + G_t. \tag{3.26}$$

G_t is assumed to follow an exogenous AR(1) process as standard in this literature.

The aggregate adopter profits is given by

$$\pi_t = \frac{\left(1+\tau_t\right) P_t^M M_t}{A_t}\left(1 - \frac{1}{\theta}\right) \tag{3.27}$$

All profits from the adopters are paid to households as a dividend every period and added to the dividends paid to them for the ownership of the firms (D_t):

$$\Pi_t = \pi_t + D_t. \tag{3.28}$$

For household preferences, I use the King-Plosser-Rebelo non-separable utitlity function

$$U\left(C_t, H_t\right) = \frac{\left(C_t^{1-\varrho}\left(1-H_t\right)^{\varrho}\right)^{1-\sigma_c} - 1}{1-\sigma_c}. \tag{3.29}$$

For technology I use a *normalized* CES function

$$Y_t = f(Z^k, K, L) = Y_0\left[\alpha\left(Z_t^k \frac{K_{t-1}}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(Z_t^l \frac{L_t}{L_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{3.30}$$

Here, following de La Grandville & Klump (2000) and Cantore et al. (2014) amongst others, Y_0, K_0, L_0 represent the point of normalization of the CES function and hence α and $1-\alpha$ can be interpreted as shares.[14]

This completes the description of the model. Section A.1 in the Appendix lists the full set of the equilibrium conditions characterizing this economy. As the model is non stationary, in order to be able to simulate it with first order perturbation methods, it needs to be stationarised. Section A.2 provide the details of the stationarization process.

[14] See Cantore & Levine (2012) for a detailed discussion about CES normalization.

3.6 Calibrated parameters

Following Cantore & Levine (2012) and Cantore et al. (2014), without loss of generality, I choose $Y_0 = 1$ at the normalization point for the CES function and I parametrise the model in order for it to coincide with the steady state. I calibrate the steady state of hours worked to be 0.33. With each period being one quarter, the discount rate, β, is set to 0.99, the depreciation rate δ to 0.025, the relative risk aversion σ_c to 2 and the capital share in the economy α to 0.33. The elasticity of substitution between capital and labour is set to 0.4, in line with available evidence for the US (Klump et al. (2012)). Unproductive government spending over GDP in steady state is calibrated to be 20% as in the national accounts for US while I assume that there is no subsidy to R&D in steady state.[15]

The weight on leisure in utility ϱ and capital stock at the normalization point/steady state K_0 are then calibrated simultaneously to match the above calibration values.

I follow closely Comin (2009), Cantore & Swarbrick (2018) and Anzoategui et al. (2016), in calibrating the parameters related to R&D and technology adoptions. I calibrate ν_1 and ν_2 to target an adoption rate $\bar{p} = 0.15/4$. This produces an average adoption lag of 7 years. The elasticity with respect to expenditures or $\partial \log(p) / \partial \log(\Gamma x) = \nu_2$ is set to $= 0.95$. I choose the mark-up in the market for materials $\theta = 1.35$ and choose χ to calibrate the average annual growth rate of output to $g^Y = 2\%$, which determines M. Finally I choose the R&D elasticity $\rho = 0.4$, the obsolescence rate $(1 - \phi) = 0.02$ and the strength of the endogenous technology mechanism $\eta = 0.5$.[16]

Regarding the R&D shock process I assume a standard deviation of 1% and an autoregressive parameter of 0.8.

3.7 Results

Figure 5 shows the impulse responses functions to a 1% increase in τ_t, the government subsidy to technology adoption. These simulations help rationalize the transmission mechanism that might generate the response in the data (Figures 2-4) trough the lenses of the model presented above.

The subsidy from the Government affects directly P_M, that is the sale of new

[15] This simplifies the steady state of the model but does not change the results. If I assume a subsidy of 15% the total amount of government spending in steady state, as in the data, the results are unchanged.

[16] η is the labour units elasticity with respect to to materials in (3.19) and it determines the strength of the endogenous persistence in the model. A higher η will generate more persistence and vice versa. A lower (higher) value of *eta* will generate less (more) persistence in the IRFs but results are qualitatively the same.

adopted technologies, which in turns increases the return to adopters. This then generates a boost in materials and, given production, firms seems to substitute capital with materials on impact. This is evident from the evolution of capital which declines substantially in the first 5 quarters since the shock. As a result, output increases but investment decreases in the first few quarters. As capital starts to accumulate again and new technologies starts to get adopted investment overshoots the initial decline and reaches a peak response after 5 quarters from the shock. This generates an hump shaped impulse response for both investment and output that is qualitatively similar to the one observed using US data.[17] The economy then moves slowly to a higher steady state in terms of output, consumption and capital. The initial decline in capital accumulation triggers a decline in hours via the complementarity in production between labour and capital income. This triggers a decline in labour income that, together with the increase in output, induces the decline in the labour share.

The only response that the model is not able to match is the one of consumption. While in the SVAR I observe the usual crowding-in effect found by Blanchard & Perotti (2002) and subsequent literature, the model predicts a substantial and persistent decline in consumption which turns positive only after about 15 quarters following the shock. This is due to the usual wealth effect present in the RBC model and it's a well known fact in the literature on fiscal policy and DSGEs (Baxter & King (1993)). Rational agents anticipate an increase in taxes following a fiscal expansion and hence decrease their consumption immediately. Various mechanism have been proposed in order to reduce or eliminate this wealth effect in DSGE models but involve the inclusion of nominal rigidities and/or other ingredients like credit constraint consumers (Galí et al. (2007)), complementarity between private and public consumption (Bouakez & Rebei (2007)) and different forms of utility (Monacelli & Perotti (2008)). While the inclusion of any of the above will probably help the model to match the response of consumption, given the aim of this work, it will unnecessarily complicate its exposition and derivations. Therefore I have decided to keep the model as simple as possible in order to be able to highlight the crucial dynamics in the transmission mechanism of the shock of interest. For the same reason I do not present simulation to a shock to *wasteful* government spending (G_t) in the model. Given the wealth effect described above the current model will not be able to match the responses observed in section 2.[18]

Summarising the findings of this section, the structural explanation for the de-

[17] Although in the data investment does not declines significantly on impact as in the model.

[18] It will generate a crowding out of consumption and investment and fiscal multiplier lower than 1 for output.

cline of the labour share following a shock to public investment in R&D seems to crucially depend on the substitutability of capital with materials and on the complementarity between capital and hours worked. Government investment in R&D generates a strong incentive for firms to substitute investment in capital structures and equipments for the adoption of new technologies. The consequent decline in capital investment together with the complementarity in production between capital and labour drives down labour income which generates the decline in the labour share.

4 Conclusions

In this paper I presented new evidence on the effects of public spending in R&D on the distribution of income across the business cycle. I started by showing how different components of fiscal spending have affected the distribution of income in the post-war US data using a SVAR model with recursive identification. Results shows that a shock to public R&D spending looks quite different from a shock to any other component of government spending. Crucially the responses of private investment and the labour income share are of opposite sign compared to the effects of other fiscal shocks. Private investment increases while the labour income share declines following an identified shock to public R&D. The boost in private investment produces a strong and long lasting positive effect on output in line with the literature on the effects of R&D on economic growth. But the negative, albeit temporary, effect on the labour share points towards a redistribution effect that favours capital income at the expenses of labour income. I then showed that the negative effect on the labour share is also robust across countries by looking at annual data for Canada, the UK, France and Germany.

In order to rationalize these results, in the second part of the paper I presented a DSGE model that can account for these stylised facts. The model includes explicitely an exogenous shock that proxies for government investment in R&D and, calibrated to the US economy, was used to shed light on the transmission mechanism of a government subsidy to R&D on investment and the labour income share as identified in the first part of the paper.

Simulations results showed how a public subsidy to the R&D process made by private firms in the model economy is able to generate simultaneously the long run positive effects on output, investment and consumption together with the short run decline in the labour share. The mechanism that generates the decline in the labour share depends on the substitutability in production between capital and adoption of new technologies together with the complementarity between capital and labour. A government subsidy to R&D induces firms to shifts resources from the accumulation of capital towards the adoption of new technologies. The consequent decline in capital accumulation produces a decline in hours worked given their complementarity with capital in the production process of firms. This in turn generates a decline in labour income that induces the fall in the labour share.

This means that while the long run effects of public investment in R&D are unambiguously positive for the economy by generating a higher level of output, investment and consumption, its short run implications can in principle generate redistributive effects that could harm agents in the economy that are heavily dependant on labour income.

Finally I want to stress that this work represents only a first step in trying to understand the effects of public spending in R&D on the redistribution of income over the business cycle. Therefore it leaves open few caveats that should be tackled in future research. The first regards the identification issues highlighted in the empirical section. Using a SVAR with recursive identification I am not able to fully address the issue of fiscal foresight here. Furthermore, the model presented here is deliberately simple and does not include nominal variables in order to try to highlight the transmission mechanism between key variables. Further research should extend it in order to study also the possible interactions with monetary policy and estimate it directly with data.

Figures

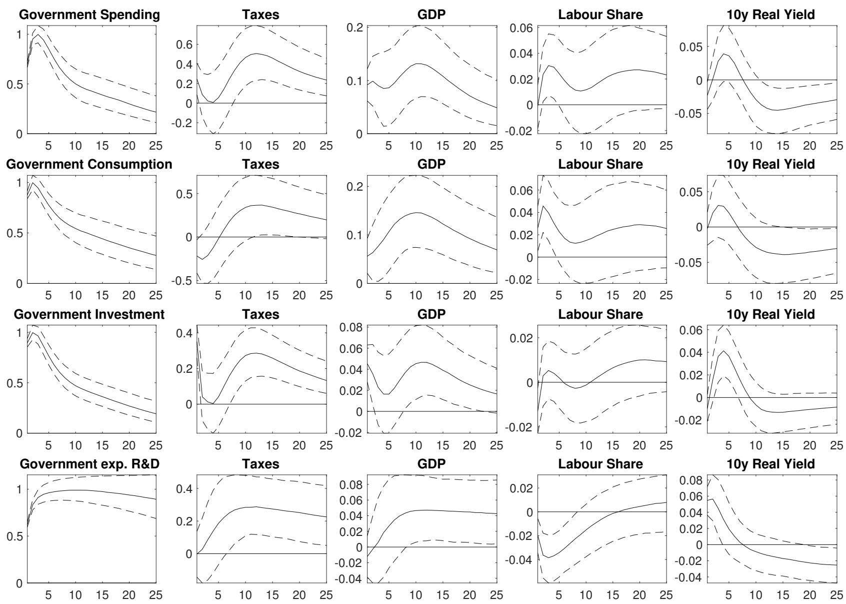

Figure 1: 5 variable SVAR responses with Choleski identification to a surprise increase in each component of Government spending in the US. Sample 1948Q1:2007Q4.

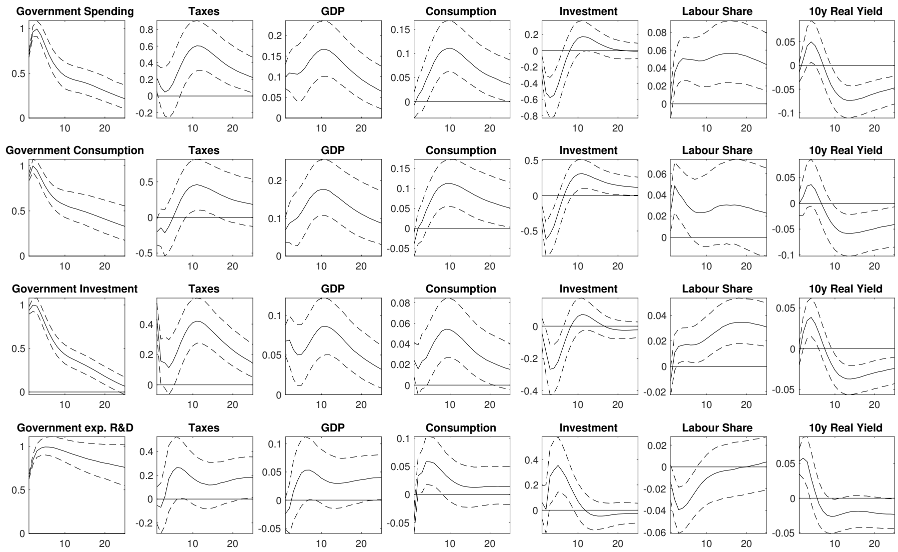

Figure 2: 7 variable SVAR responses with Choleski identification to a surprise increase in each component of Government spending in the US. Sample 1948Q1:2007Q4.

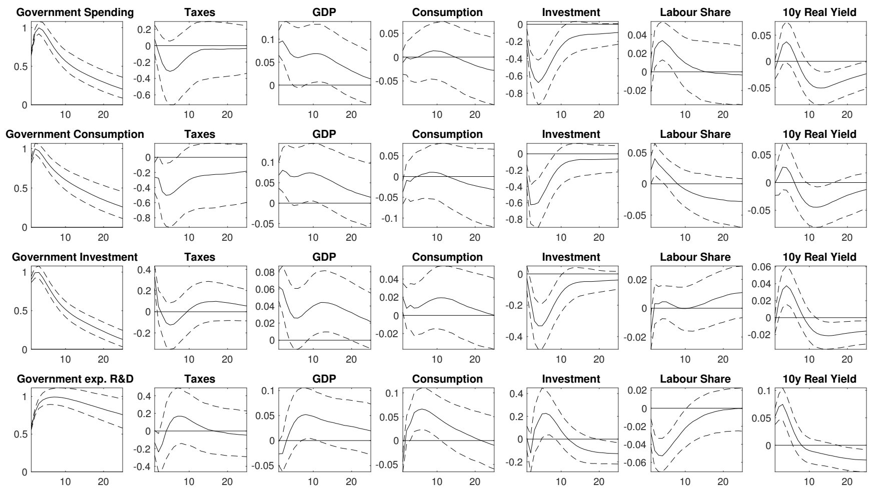

Figure 3: 7 variable SVAR responses with Choleski identification to a surprise increase in each component of Government spending in the US. Sample 1948Q1:2016Q4.

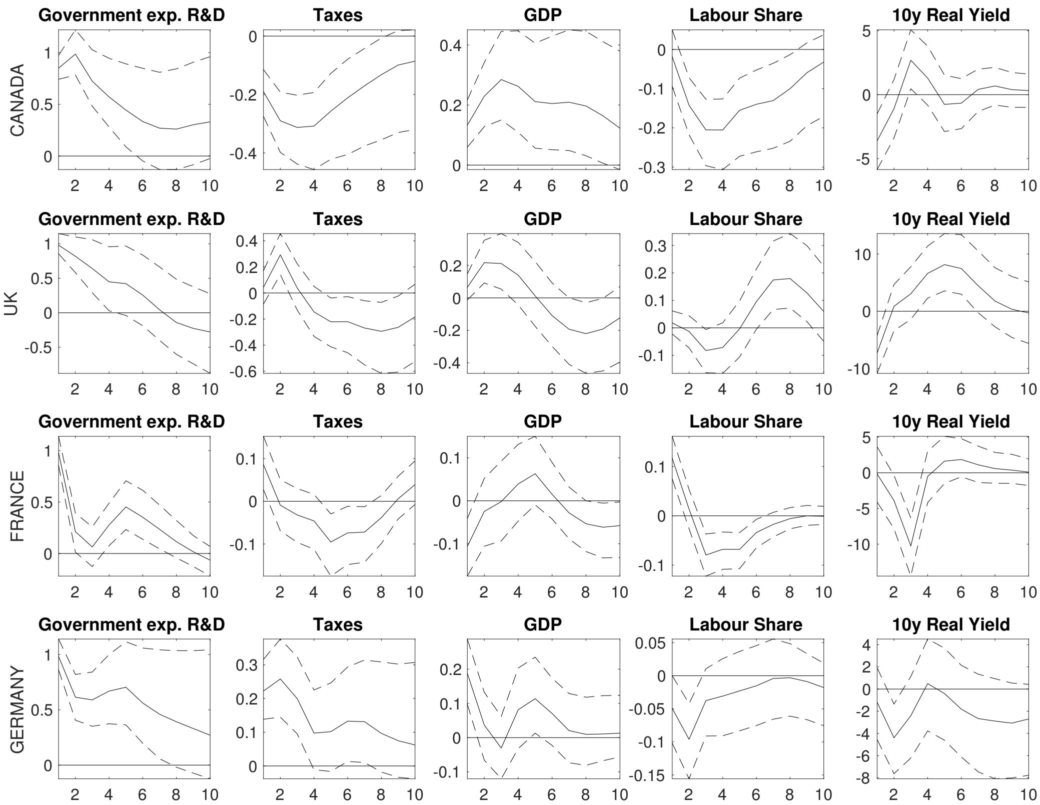

Figure 4: 5 variable SVAR responses with Choleski identification to a surprise increase in each component of Government spending using OECD cross country Annual data.

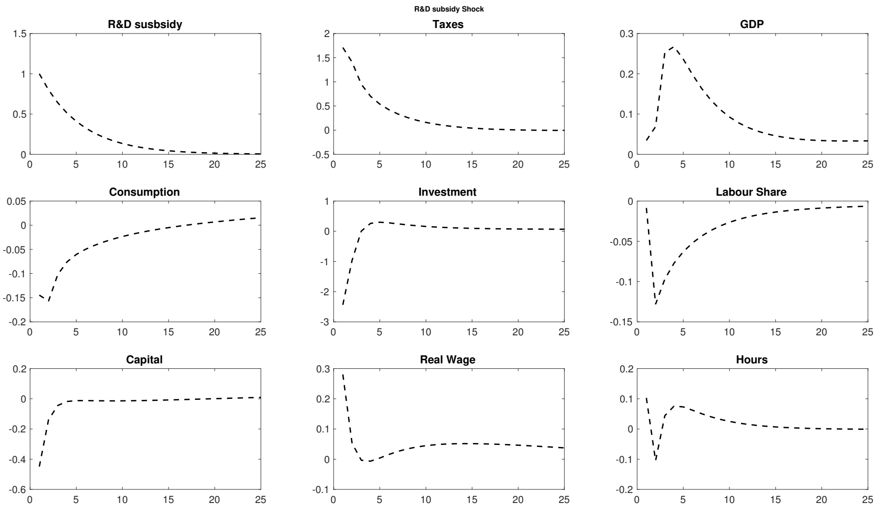

Figure 5: Model impulse responses functions to a 1% increse in the public R&D subsidy.

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A Appendix

A.1 Equilibrium Conditions

Equilibrium conditions for $\{\ H_t,\ K_t,\ W_t,\ Z_t,\ Q_t,\ C_t,\ T_t\ \}$:

$$\frac{U'(H_t)}{U'(C_t)} = -W_t \tag{A.1}$$

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1}\left(r_{t+1} + (1-\delta)\right)\right] \tag{A.2}$$

$$\hat{W}_t = f'(L_t) \tag{A.3}$$

$$r_t = f'(K_{t-1}) \tag{A.4}$$

$$K_t = I_t + (1-\delta)K_{t-1} \tag{A.5}$$

$$Y_t = C_t + I_t + S_t + x_t\left(Z_{t-1} - A_{t-1}\right) + T_t \tag{A.6}$$

$$T_t = \tau_t P_t^M M_t + G_t \tag{A.7}$$

$$\tag{A.8}$$

The additional equations for the R&D sector variables A_t, Z_t, p_t, φ_t, v_t, w_t, x_t, S_t, P_t^M, M_t, Γ_t are:

$$A_t = p_t\phi\left[Z_{t-1} - A_{t-1}\right] + \phi A_{t-1} \tag{A.9}$$

$$Z_t = Z_{t-1}\left[\chi\left(\frac{S_t}{K_t}\right)^{\rho} + \phi\right] \tag{A.10}$$

$$v_t = \frac{(1+\tau_t)\,P_t^M M_t}{A_t}\left(1 - \frac{1}{\theta}\right) + \phi\mathbb{E}_t\left[\Lambda_{t,t+1}v_{t+1}\right] \tag{A.11}$$

$$w_t = -x_t + \phi\mathbb{E}_t\left[\Lambda_{t,t+1}\left(p_t\left(\Gamma_t x_t\right)v_{t+1} + \left[1 - p_t\left(\Gamma_t x_t\right)\right]w_{t+1}\right)\right] \tag{A.12}$$

$$1 = \phi\mathbb{E}_t\left[\Lambda_{t,t+1}p_t'\left(\Gamma_t x_t\right)\left(v_{t+1} - w_{t+1}\right)\Gamma_t\right] \tag{A.13}$$

$$S_t = \mathbb{E}_t\left[\Lambda_{t,t+1}w_{t+1}\left(Z_t - \phi Z_{t-1}\right)\right] \tag{A.14}$$

$$(1+\tau_t)\,P_t^M = A_t^{\frac{1-\theta}{\theta}}\theta \tag{A.15}$$

$$\frac{\eta}{1-\eta}\mathbb{E}_t\left[\Lambda_{t,t+1}W_{t+1}H_{t+1}\right] = \mathbb{E}_t\left[\Lambda_{t,t+1}P_t^M M_t\right] \tag{A.16}$$

$$\Gamma_t = \frac{A_t}{K_t} \tag{A.17}$$

With functional forms, FOCs and shocks processes:

$$\frac{U'(H_t)}{U'(C_t)} = -\frac{\varrho}{1-\varrho}\frac{C_t}{1-H_t} \tag{A.18}$$

$$U'(C_t) = (1-\varrho)\left(C_t\right)^{(1-\varrho)(1-\sigma_c)-1}\left(1-H_t\right)^{\varrho(1-\sigma_c)} \tag{A.19}$$

$$f'(L_t) = (1-\alpha)\left(\frac{Y_0}{L_0}Z_t^l\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}} \tag{A.20}$$

$$f'\left(K_t\right) = \alpha \left(\frac{Y_0}{K_0}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_{t-1}}\right)^{\frac{1}{\sigma}} \tag{A.21}$$

$$L_t = M_{t-1}^{\eta}\left(H_t\right)^{1-\eta} \tag{A.22}$$

$$W_t = \hat{W}_t\left(1-\eta\right)\left(\frac{M_{t-1}}{H_t}\right)^{\eta} \tag{A.23}$$

$$log(Z_t^l) = \rho^{Zl} log(Z_{t-1}^l) + \varepsilon_t^{Zl} \tag{A.24}$$

$$log(Z_t^k) = \rho^{Zu} log(Z_{t-1}^k) + \varepsilon_t^{Zk} \tag{A.25}$$

$$log(\tau_t) = \rho^{\tau} log(\tau_{t-1}) + \varepsilon_t^{\tau} \tag{A.26}$$

$$log(G_t) = \rho^{G} log(G_{t-1}) + \varepsilon_t^{G} \tag{A.27}$$

where $\epsilon_t^i \sim \mathcal{N}\left(0, \sigma_i^2\right)$ for $i = Zk, Zl, \tau, G$.

A.2 Stationary Equilibrium

As the model is nonstationary, I adjust the conditions for simulation. Note that household labour supply is stationary, and so I can define

$$\tilde{L}_t = \frac{L_t}{M_{t-2}^{\eta}} = \left(1+g_{t-1}^M\right)^{\eta}\left(H_t\right)^{1-\eta} \tag{A.28}$$

as stationarised efficiency labour, where g_t^M is the growth rate of technical progress M_t. Then

$$\tilde{Y}_t = \frac{Y_t}{M_{t-2}^{\eta}} = Y_0\left[\alpha\left(Z_t^k\frac{\tilde{K}_{t-1}}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(Z_t^l\frac{\tilde{L}_t}{L_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{A.29}$$

where $\tilde{K}_t = K_t/M_{t-1}^{\eta}$. The efficiency wage is stationary, but the labour wage rate can be stationarised with

$$\tilde{W}_t = \frac{W_t}{M_{t-2}^{\eta}} = \hat{W}_t\left(1-\eta\right)\left(\frac{1+g_{t-1}^M}{H_t}\right)^{\eta} \tag{A.30}$$

where $\tilde{W}_t = W_t/M_{t-2}^{\eta}$. The demand for materials can then be written

$$\frac{\eta}{1-\eta}\mathbb{E}_t\left[\Lambda_{t,t+1}\tilde{W}_{t+1}H_{t+1}\right] = \mathbb{E}_t\left[\Lambda_{t,t+1}\tilde{P}_t^M\left(1+g_t^M\right)\right] \tag{A.31}$$

where $\tilde{P}_t^M \equiv P_t^M/M_{t-1}^{\eta-1}$. Then

$$\tilde{P}_t^M = \left(\tilde{A}_t\right)^{\frac{1-\theta}{\theta}}\frac{\theta}{(1+\tau_t)} \tag{A.32}$$

$\tilde{A}_t \equiv A_t/M_{t-1}^{(1-\eta)\frac{\theta}{\theta-1}}$. Letting

$$g_t^a \equiv \left(1+g_{t-1}^M\right)^{(1-\eta)\frac{\theta}{\theta-1}} - 1 = \frac{1+g_t^A}{1+g_t^{\tilde{A}}} - 1, \tag{A.33}$$

be the growth rate of A_t relative to that of the stationary $\tilde{A}_t$, these definitions lead to

$$\tilde{A}_t\left(1+g^a_{t-1}\right)=p_t\phi\left[\tilde{Z}_{t-1}-\tilde{A}_{t-1}\right]+\phi\tilde{A}_{t-1} \tag{A.34}$$

$$\tilde{Z}_t\left(1+g^a_{t-1}\right)=\tilde{Z}_{t-1}\left[\chi\left(\frac{\tilde{S}_t}{\tilde{K}_t}\right)^{\rho}+\phi\right] \tag{A.35}$$

where Z_t is scaled by the same factor as A_t and with $\tilde{S}_t\equiv S_t/M^{\eta}_{t-1}$. I can then complete the R&D sector and write

$$\tilde{S}_t\left(1+g^a_{t-1}\right)=\mathbb{E}_t\left[\Lambda_{t,t+1}\tilde{w}_{t+1}\left(\tilde{Z}_t\left(1+g^a_{t-1}\right)-\phi\tilde{Z}_{t-1}\right)\right] \tag{A.36}$$

$$\tilde{v}_t=\frac{(1+\tau_t)\,\tilde{P}^M_t\left(1+g^M_t\right)}{\tilde{A}_t}\left(1-\frac{1}{\theta}\right)+\left(1+g^X_t\right)\phi\mathbb{E}_t\left[\Lambda_{t,t+1}\tilde{v}_{t+1}\right] \tag{A.37}$$

$$\tilde{w}_t=-\tilde{x}_t+\left(1+g^X_t\right)\phi\mathbb{E}_t\left[\Lambda_{t,t+1}\left(p_t\left(\frac{\tilde{A}_t}{\tilde{K}_t}\tilde{x}_t\right)\tilde{v}_{t+1}+\left[1-p_t\left(\frac{\tilde{A}_t}{\tilde{K}_t}\tilde{x}_t\right)\right]\tilde{w}_{t+1}\right)\right] \tag{A.38}$$

$$1=\left(1+g^X_t\right)\phi\mathbb{E}_t\left[\Lambda_{t,t+1}p'_t\left(\frac{\tilde{A}_t}{\tilde{K}_t}\tilde{x}_t\right)\left(\tilde{v}_{t+1}-\tilde{w}_{t+1}\right)\frac{\tilde{A}}{\tilde{K}_t}\right] \tag{A.39}$$

where $\tilde{w}_t$, $\tilde{v}$ and $\tilde{x}_t$ (and $\tilde{\Gamma}_t=\tilde{A}_t/\tilde{K}_t$) are scaled by $M_{t-1}^{\eta-(1-\eta)\frac{\theta}{\theta-1}}$ and

$$g^X_t\equiv\left(1+g^M_{t-1}\right)^{\eta-(1-\eta)\frac{\theta}{\theta-1}}-1=\frac{1+g^x_t}{1+g^{\tilde{x}}_t}-1. \tag{A.40}$$

is the growth rate of x_t, w_t and v_t relative to that of the stationary $\tilde{x}_t$, $\tilde{w}_t$ and $\tilde{v}_t$ respectively. Scaling consumption, investment, and government spending in the remainder of the model by the same factor as output yields a stationary model. The only amended first order condition are the Euler equation, resource constraint and capital law of motion. Given the functional form of preferences:

$$U_t=\frac{\left(C_t^{(1-\varrho)}\left(1-H_t\right)^{\varrho}\right)^{1-\sigma_c}-1}{1-\sigma_c} \tag{A.41}$$

this leads to

$$1=\mathbb{E}_t\left[\beta\frac{U'\left(\tilde{C}_{t+1}\right)}{U'\left(\tilde{C}_t\right)}\left(1+g^y_{t-1}\right)^{(1-\varrho)(1-\sigma_c)-1}\left(\alpha\gamma\frac{\tilde{Y}_{t+1}}{\tilde{K}_t}+(1-\delta)\right)\right] \tag{A.42}$$

$$\tilde{K}_t\left(1+g^y_{t-1}\right)=\tilde{I}_t+(1-\delta)\tilde{K}_{t-1} \tag{A.43}$$

$$\tilde{Y}_t=\tilde{C}_t+\tilde{I}_t+\tilde{S}_t+\tilde{x}_t\left(\tilde{Z}_{t-1}-\tilde{A}_{t-1}\right)\left(1+g^X_{t-1}\right) \tag{A.44}$$

where

$$g^y_t\equiv\left(1+g^M_t\right)^{\eta}-1=\frac{1+g^Y_t}{1+g^{\tilde{Y}}_t}-1 \tag{A.45}$$

The tildes are dropped henceforth.

A.3 Steady State

The steady state of the stationary model[19] can be found as follows:

$$Z^k = Z^l = \tau = G = 1 \tag{A.46}$$

$$Q = 1 \tag{A.47}$$

$$Y = Y_0 \tag{A.48}$$

$$H = H_0 \tag{A.49}$$

$$r = \frac{1}{\hat{\beta}} - 1 + \delta \tag{A.50}$$

$$K = K_0 = \frac{\alpha Y}{r} \tag{A.51}$$

$$g^Y = 0.02/4 \tag{A.52}$$

$$L = L_0 = (1 + g^Y)\,(H_0)^{1-\eta} \tag{A.53}$$

where $\hat{\beta}$ is the growth adjusted discount factor. Then using the income shares identity at the normalization point:

$$\hat{W} = (1-\alpha)\frac{Y_0}{L_0}. \tag{A.54}$$

$$\tag{A.55}$$

The rest of the steady states relationship follow trivially from here.

[19]Note that I am dropping the tildes defined in previous section.